

# The (2+1)-dimensional gravastars

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## Abstract

We propose a new model of a *gravastar* in (2+1) anti-de Sitter space-time. This new three dimensional configuration has three different regions with different equations of state: [I] Interior:  $0 \leq r < r_1$ ,  $\rho = -p$ ; [II] Shell:  $r_1 \leq r < r_2$ ,  $\rho = p$ ; [III] Exterior:  $r_2 < r$ ,  $\rho = p = 0$ . The outer region of this gravastar corresponds to the exterior (2 + 1) anti-de Sitter space-time, popularly known as the BTZ space-time. Like BTZ model,  $\Lambda$  is taken to be negative, which at the junction turns out to be positive as required by stability of gravastar and mathematical consistency. After investigating the Interior space-time, Shell and Exterior space-time we have highlighted different physical features in terms of Length and Energy, Entropy, and Junction conditions of the spherical distribution. It is shown that the present model of charge-free gravastar in connection to the exterior (2+1) anti-de Sitter space-time or the BTZ space-time is non-singular.

*Key words:* Gravitation; Equation of State; Gravastar

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## 1 Introduction

Very recently, we proposed a charged (3+1)-dimensional gravastar admitting conformal motion [1] in the frame work of Mazur and Mottola model of a chargeless gravastar [2,3]. The model implies that the space of a gravastar has three different regions defined by different equations of state (EOS) as: (i) interior:  $0 < r \leq r_1$ ,  $p = -\rho$ , (ii) shell:  $0 < r_1 < r < r_2$ ,  $p = +\rho$  and (iii) exterior:  $r_2 < r$ ,  $p = \rho = 0$ . This provides an alternative to static black holes, however, energy density is found to diverge in the interior region of the gravastar, which scales like an inverse second power of its radius. This behaviour makes the model singular at  $r = 0$ .

In order to find a non-singular alternative to black holes, it is worth exploring non-singular gravastars in the (2+1)-dimensions which offer clarity to study concepts of gravity. Thus, we study the work of Mazur and Mottola with chargeless gravastar in (3+1)-dimensions [2,3] under the (2+1)-dimensional space-time. We show that the (2+1)-dimensional neutral gravastars do exist without any curvature singularity at the origin, which may be considered as an alternative to BTZ black holes as presented by Bañados, Teitelboim and Zanelli (BTZ) [4].

It is known that in (2+1) dimensional space-time, Newtonian theory can not be obtained as a limit of Einstein's theory. In other words, general relativity in (2+1) dimensions has no Newtonian limit and no propagating degrees of freedom [5]. Also, gravastar structure can not be obtained from Newtonian gravity. But it is argued that Einstein's general theory of relativity admits gravastar structure that has three different regions such as interior, shell and exterior defined by different equations of state (EOS) and this provides an alternative to static black holes. The presence of matter on the thin shell is required to achieve the crucial stability of such systems under expansion by exerting an inward force to balance the repulsion from within. For this reason, (2+1) dimensional gravastar structure have considered here.

Collapsing matter may appear into a final state of (2+1)-dimensional black hole with no curvature singularity at the origin. An explicit vacuum solution of (2+1)-dimensional gravity with nonzero negative cosmological constant obtained by Bañados, Teitelboim and Zanelli (BTZ) [4] did demonstrate the existence of such physical systems with an event horizon and thermodynamic properties similar to (3+1) dimensional black holes. These are known as BTZ black holes, which are asymptotically anti-de Sitter. A great number of good reviews on the subject are available in the literature [6,7,8].

For a (2+1)-dimensional gravastar, intermediate region is a 2-dimensional junction instead of a closed spherical shell. Like BTZ model,  $\Lambda$  is taken to

be negative, which at the junction shell turns out to be positive as required for gravastar and mathematical consistency. It is like requirement of negative energy at the junction between two separated gravitational sources to cancel out excess positive energy. The scheme of the investigation is as follows: In Section II, III and IV respectively we discuss the Interior space-time, Exterior space-time and Shell whereas sections V, VI and VII respectively are devoted to the Length and Energy, Entropy, and Junction conditions of the spherical distribution.

## 2 Interior space-time

We take the line element for the interior space-time of a static spherically symmetric distribution of matter in  $(2 + 1)$  dimensions in the form

$$ds^2 = -e^{2\gamma(r)} dt^2 + e^{2\mu(r)} dr^2 + r^2 d\theta^2. \quad (1)$$

We assume that the matter distribution at the interior of the star is perfect fluid type, given by

$$T_{ij} = (\rho + p)u_i u_j + p g_{ij}, \quad (2)$$

where,  $\rho$  represents the energy density,  $p$  is the isotropic pressure, and  $u^i$  is the 3-velocity of the fluid. The Einstein's field equations with a cosmological constant ( $\Lambda < 0$ ), for the space-time described by the metric (1) together with the energy-momentum tensor given in Eq. (2), yield (rendering  $G = c = 1$ )

$$\frac{\mu' e^{-2\mu}}{r} = 2\pi\rho + \Lambda, \quad (3)$$

$$\frac{\gamma' e^{-2\mu}}{r} = 2\pi p - \Lambda, \quad (4)$$

$$e^{-2\mu} (\gamma'^2 + \gamma'' - \gamma' \mu') = 2\pi p - \Lambda, \quad (5)$$

where a ‘ $\prime$ ’ denotes differentiation with respect to the radial parameter  $r$ . Combining Eqs. (3)-(5), we get

$$(\rho + p) \gamma' + p' = 0, \quad (6)$$

which is the conservation equation in  $(2 + 1)$  dimensions.

In the interior region I, by the use of the assumption  $p = -\rho$  iteratively, the equation (6) yields

$$\rho = \text{constant} = \rho_c, \quad (\text{say}). \quad (7)$$

Let us call this constant as,  $\rho_c = H_0^2/2\pi$  where  $H_0$  is the Hubble parameter with its present day constant value. In other words, it may be expressed as follows

$$p = -\rho_c. \quad (8)$$

We would like to mention that the equation of state  $p = -\rho$  (known in the literature as a false vacuum, degenerate vacuum, or  $\rho$ -vacuum [9,10,11,12]) represents a repulsive pressure. In the context of an accelerating Universe, it is argued by Usmani et al. [1] that the equation of state of the kind  $p = -\rho$  may be related to the  $\Lambda$ -dark energy, an agent responsible for the second phase of the inflation of Hot Big Bang theory [13,14,15,16,17].

By using the Eq. (8), one can get the solutions for  $\gamma$  and  $\mu$  from the field equations as

$$e^{2\gamma} = e^{-2\mu} = A + (-\Lambda - 2\pi\rho_c)r^2, \quad (9)$$

where A is an integration constant. Hence, the active gravitational mass  $M(r)$  can be expressed at once in the following form

$$M(r) = \int_0^R 2\pi r \rho dr = \pi \rho_c R^2. \quad (10)$$

From the above Eq. (9) we observe that the space-time metric thus obtained is free from any central singularity.

### 3 Exterior space-time

For the exterior region ( $p = \rho = 0$ ), the BTZ space-time is

$$ds^2 = - \left( -M_0 - \Lambda r^2 \right) dt^2 + \left( -M_0 - \Lambda r^2 \right)^{-1} dr^2 + r^2 d\theta^2. \quad (11)$$

The parameter  $M_0$  is the conserved mass associated with asymptotic invariance under time displacements. This mass is given by a flux integral through a large circle at space-like infinity.

## 4 Shell

The equation of state in the shell II is  $p = \rho$ , which represents a stiff fluid. We note that this type of equation of state which refers to a Zel'dovich Universe have been selected by several authors for various situations in cosmology [18,19,20] as well as astrophysics [21,22,23].

Within the shell the Eqs. (3) - (5) can be solved implicitly. However, it is possible to obtain analytic solution in the thin shell limit,  $0 < e^{-2\mu} \equiv h \ll 1$ . We first try to find implicit solution and later we shall find limiting solution.

### 4.1 General case

By equating Eqs. (4) and (5), one gets

$$\gamma'^2 + \gamma'' - \gamma'\mu' = \frac{\gamma'}{r}$$

which eventually yields

$$e^\mu = \frac{e^\gamma \gamma'}{r}. \quad (12)$$

After a little bit manipulation and calculation, we then get

$$r\gamma'' + 2\Lambda e^{2\gamma}\gamma'^3 + \gamma' = 0. \quad (13)$$

By the use of any computer programme one can get a solution which, after exploiting the stiff fluid equation of state  $p = \rho$  and the Eq. (6), is as follows:

$$p = p_0 e^{-2\gamma}, \quad (14)$$

$p_0$  being an integration constant. Thus, if  $\gamma$  is known, then all parameters can be found in terms of  $\gamma$ .

#### 4.2 Thin shell limit

Let us define  $e^{-2\mu} \equiv h(r)$ . Then the field Eqs. (3) -(6), with  $p = \rho$ , may be recast in the forms

$$2\Lambda = -\frac{h'}{2r} - \frac{1}{r}\gamma'h, \quad (15)$$

$$h = \frac{r^2}{[(e^\gamma)']^2}. \quad (16)$$

Now, it is possible to obtain an analytical solution in the thin shell limit,  $0 < e^{-2\mu} \equiv h \ll 1$ . In this limit, we can set  $h$  to zero on the right hand side of (15) leading to order and integrate immediately to yield

$$h = B - 2\Lambda r^2, \quad (17)$$

where  $B$  is an integration constant.

The other unknown functions are

$$e^{2\gamma} = \left[ C - \frac{1}{2\Lambda} \sqrt{B - 2\Lambda r^2} \right]^2. \quad (18)$$

Also, using (14), one can get

$$p = \rho = p_0 \left[ C - \frac{1}{2\Lambda} \sqrt{B - 2\Lambda r^2} \right]^{-2}, \quad (19)$$

$C$  being an integration constant.

For consistency of this thin shell limit solutions, we have the constant pressure term as

$$p_0 = \frac{\Lambda_0 C^2}{2\pi}, \quad (20)$$

where  $\Lambda_0$  is the Einstein's erstwhile cosmological constant.

### 5 Proper length and Energy

We assume the interfaces at  $r = R$  and  $r = r_2$  describing the phase boundary of region I and region III respectively are very close. That is,  $r_2 = R + \epsilon$  with

$0 < \epsilon \ll 1$ . The proper thickness between two interfaces i.e. of the shell is obtained as:

$$\ell = \int_R^{r_2} \sqrt{e^{2\mu}} dr = \frac{1}{\sqrt{2\Lambda_0}} \left[ \sin^{-1} \left( \frac{r\sqrt{2\Lambda_0}}{\sqrt{B}} \right) \right]_R^{R+\epsilon}. \quad (21)$$

If we expand  $F(R + \epsilon)$  binomially about  $R$  and take first order of  $\epsilon$ , then,  $F(R + \epsilon) \simeq F(R) + \epsilon F'(R)$  and our  $\ell$  would be  $\ell = \epsilon / \sqrt{B - 2\Lambda_0 R^2}$ . Obviously, here the length must be a real and positive quantity and the constraint should be  $B > 2\Lambda_0 R^2$ . Actually, the thickness between two interfaces becomes infinite for the constant value of  $B$  as  $B = 2\Lambda_0 R^2$  and hence this identity is not applicable.

We now calculate the energy  $\tilde{E}$  within the shell only and we get

$$\tilde{E} = 2\pi \int_R^{R+\epsilon} \rho r dr = \left[ \frac{8\pi\Lambda_0^2 C}{p_0(\sqrt{B - 2\Lambda_0 r^2} - 2\Lambda_0 C)} - \frac{4\pi\Lambda_0}{p_0} \ln(2\Lambda_0 C - \sqrt{B - 2\Lambda_0 r^2}) \right]_R^{R+\epsilon}. \quad (22)$$

In the similar way, as we have done above, by expanding  $F(R + \epsilon)$  binomially about  $R$  and taking first order of  $\epsilon$ , we get  $\tilde{E} = \epsilon R / [C - \frac{1}{2\Lambda_0} \sqrt{B - 2\Lambda_0 R^2}]^2$  with the condition to be imposed  $B > 2\Lambda_0 R^2$ .

## 6 Entropy

Following Mazur and Mottola [2,3], we now calculate the entropy by letting  $r_1 = R$  and  $r_2 = R + \epsilon$ :

$$S = 2\pi \int_R^{R+\epsilon} s(r) r \sqrt{e^{2\mu}} dr. \quad (23)$$

Here  $s(r)$  is the entropy density for the local temperature  $T(r)$ , which may be written as

$$s(r) = \frac{\alpha^2 k_B^2 T(r)}{4\pi \hbar^2} = \alpha \left( \frac{k_B}{\hbar} \right) \sqrt{\frac{p}{2\pi}}, \quad (24)$$

where  $\alpha^2$  is a dimensionless constant.

Thus the entropy of the fluid within the shell, via the Eq. (24), becomes

$$\begin{aligned}
S &= 2\pi \int_R^{R+\epsilon} \alpha \left( \frac{k_B}{\hbar} \right) \frac{1}{\sqrt{2\pi p_0}} \frac{r dr}{\left[ \sqrt{B - 2\Lambda r^2} \left( C - \frac{1}{2\Lambda} \sqrt{B - 2\Lambda r^2} \right) \right]} \\
&= \frac{2\pi \alpha k_B}{\hbar \sqrt{2\pi p_0}} \left[ \ln |2\Lambda C - \sqrt{B - 2\Lambda r^2}| \right]_R^{R+\epsilon}. \quad (25)
\end{aligned}$$

## 7 Junction Condition

The discontinuity in the extrinsic curvature determine the surface stress energy and surface tension of the junction surface at  $r = R$ . Here the junction surface is a one dimensional ring of matter. Let,  $\eta$  denotes the Riemann normal coordinate at the junction. We assume  $\eta$  be positive in the manifold in region III described by exterior BTZ spacetime and  $\eta$  be negative in the manifold in region I described by our interior space-time and  $x^\mu = (\tau, \phi, \eta)$ .

The second fundamental forms associated with the two sides of the shell [24,29,27,25,26,28] are given by

$$K_j^{i\pm} = \frac{1}{2} g^{ik} \frac{\partial g_{kj}}{\partial \eta} \Big|_{\eta=\pm 0} = \frac{1}{2} \frac{\partial r}{\partial \eta} \Big|_{r=R} g^{ik} \frac{\partial g_{kj}}{\partial r} \Big|_{r=R}. \quad (26)$$

So, the discontinuity in the second fundamental forms is given as

$$\kappa_{ij} = K_{ij}^+ - K_{ij}^-. \quad (27)$$

In (2+1) dimensional spacetime, the field equations are derived [29]:

$$\sigma = -\frac{1}{8\pi} \kappa_\phi^\phi, \quad (28)$$

$$v = -\frac{1}{8\pi} \kappa_\tau^\tau, \quad (29)$$

where  $\sigma$  and  $v$  are line energy density and line tension. Employing relevant information into Eqs. (28) & (29) and setting  $r = R$ , we obtain

$$\sigma = -\frac{1}{8\pi R} \left[ \sqrt{-\Lambda R^2 - M_0} + \sqrt{A + (-\Lambda - 2\pi\rho_c)R^2} \right], \quad (30)$$

$$v = -\frac{1}{8\pi} \left[ \frac{-\Lambda R}{\sqrt{-\Lambda R^2 - M_0}} + \frac{(-\Lambda - 2\pi\rho_c)R}{\sqrt{A + (-\Lambda - 2\pi\rho_c)R^2}} \right]. \quad (31)$$



One can note that the line tension is negative which implies that there is a line pressure as opposed to a line tension. As expected in (3+1) dimensional case the energy density is negative in the junction shell. In our configuration, the thin shell i.e. region II contains ultra-relativistic fluid obeying  $p = \rho$  as well as discontinuity of second fundamental form provides some extra surface stress energy and surface tension of the junction interface. These two non-interacting components of the stress energy tensors are characterizing features of our non-vacuum region II.

## 8 Conclusion

In the neutral gravastar in connection to the exterior  $(2 + 1)$  anti-de Sitter space-time (asymptotically the BTZ space-time) we have presented a stable and non-singular model. The three dimensional configuration of this model has three different regions with different equations of state: an Interior with geometric and physical structure  $0 \leq r < r_1$ ,  $\rho = -p$ ; a Shell with  $r_1 \leq r < r_2$ ,  $\rho = p$ , and an Exterior with  $r_2 < r$ ,  $\rho = p = 0$ .

However, to obtain a realistic picture, following BTZ model  $\Lambda$  is taken to be negative, which at the junction turns out to be positive as required for the structure of gravastar. As mentioned in the Introductory part that Bañados, Teitelboim and Zanelli [4] have obtained a unique black hole solution in the form of point mass with negative cosmological constant that has a horizon and the radius of curvature  $= \sqrt{(-\Lambda)}$  provides the necessary length scale. However, for  $\Lambda \geq 0$  a cosmological type horizon exist with a naked singularity. The concept of gravastar is to search configuration which is alternative to black hole. In this paper, we propose the gravastar configuration which is alternative to BTZ black hole. We have investigated all these regions and highlighted different physical features in terms of Length and Energy, Entropy, and Junction conditions of the spherical distribution.

In this regard we would like to mention that though both the present work and the one by Usmani et al. [1] are based on the idea of the Mazur-Mottola model but they differ in two aspects: here conformal motion as well as  $3 + 1$  space-time have not been adopted. Also, here instead of charge we have considered a neutral spherical system. Under the  $2 + 1$  dimensional geometry and non-charged physical structure we have obtained a non-singular solution for gravastar. However, this demands that one should investigate a  $2 + 1$  dimensional solution of charged gravastar.

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